Simple Model of Nuclear Interaction

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In this paper, the atoms of the periodic table are defined as independent particles. Atomic nuclei are considered special quantized oscillators (mass oscillators), which have specific ground and excited states. The frequency of nuclear oscillators is proportional to the square root of the mass of the nucleus. The rest mass of atomic nuclei is determined using Planck's black body radiation theory. The Radiation Model of atomic nuclei is determined by two parameters, the mass of the neutron and a dimensionless number: *Q = 2 / 9.* Nuclear reactions are interpreted with the excitation levels of the atomic-nuclear oscillators.

Keywords: nuclear interaction, single-particle model of the atomic nucleus, strong interaction, weak interaction, mass oscillator, Planck's black body radiation theory, modeling of nuclear reactions

1. Introduction

According to our current knowledge, the atomic nucleus is a bound state of protons and neutrons (nucleons), which is held together by the nuclear interaction. In the 1960s, the possibility arose that nucleons could be further broken down into quark particles. The force that holds quarks together also holds atomic nuclei together, this is the strong force of interaction, the theory of which is Quantum Chromodynamics (QCD). The currently generally accepted QARK-QCD theory of atomic nuclei is a multiparticle theory, the description of the atomic nucleus can be done using the well-known mathematical formalism of quantum mechanics. Based on the new theory, no information can be found in the literature about only one successful binding energy calculations, even for the simplest nucleus, deuterium.

To this day, successful mass calculations related to atomic nuclei are based on the one-particle model of the nucleus, the liquid drop model of the nucleus, born in the 1930s [3].

In this work, we also provide the one-particle nuclear model recently developed by the Author, which is based on the thermal radiation of high-temperature stars. The ground state binding energy of atomic nuclei is provided by electromagnetic and neutrino radiation emitted by stars. The newly introduced one-particle nuclear model, similarly to the old nuclear droplet model, can calculate only the ground state mass of the atoms without their nuclear spin.

2. The Nuclear Radiation Model

In this paper each atomic nucleus is considered as a single particle, which has both its ground state and its excited states. Each nucleus is a quantized oscillator ("mass oscillator" for short), the frequency of which is proportional to the square root of the mass of the nucleus. In very high-temperature astronomical objects, atomic nuclei are born, inside the object there are excited and ground state nuclei. The resulting nuclei emit strong gamma radiation and neutrino radiation, which provide the binding energy of the nuclei.

High-temperature radiation is assumed to follow Planck's blackbody radiation law. Each atomic nucleus acts as a black body oscillator, the radiation frequency of the atoms is proportional to the square root of the mass of the nucleus.

In a good approximation, the radiation frequency of the nucleus with mass number *A*

$$\omega_{Z,A} \propto \sqrt{M(Z,A)} \propto \sqrt{A}. \tag{1}$$

Planck's black body radiation law for discrete radiation frequencies

$$E_{Z,A}^{rad} \propto \frac{\omega_{Z,A}^4}{\exp(\hbar\omega_{Z,A} / kT) - 1}. \tag{2}$$

This law was used to calculate the binding energy of each atomic nucleus.

The Radiation Model of atomic nuclei can be written in a similar form to the long-known nuclear liquid drop model. Here, based on the Radiation Model, we write down the ground state masses of individual nuclei (atoms)

$$M(Z,A) = M_0(A) + M_{rad}(A) + M_{as}(Z,A) + M_p(Z,A), \tag{3}$$

The first term is the "base mass"

$$M_0(A) = (1-\alpha_2)AM_N(fit);\ \left(M_N(fit) \cong M_N\right), \tag{4}$$

where α_2 and $M_N(fit)$ are fitting parameters, M_N denotes the mass of the neutron. The radiation term is next

$$M_{rad}(A) = -\alpha_4 \frac{\omega^4(A)}{B^{\omega} - 1} \equiv -\alpha_4 \frac{A^2 M_0^2}{B^{\sqrt{AM_0}} - 1} \equiv -\alpha_4 \frac{A^2 M_0^2}{R(A)}, \tag{5}$$

where α_4 and *B* are fitting parameters. The last member of the formula needs explanation; where the denominator is formally the "gravitational radius". The reason for the name is simple, the radiation term fully corresponds to the gravitational self-energy, which is proportional to the square of the atomic mass (gravitational part) and inversely proportional to the gravitational radius of the atom.

The asymmetry term of the mass formula, which is proportional to the square of the proton-neutron number, α_0 is a fitting parameter

$$M_{as}(Z,A) = \alpha_0 M_0^2 \left(\frac{A-2Z}{A+3}\right)^2, \tag{6}$$

Finally, the pair energy term

$$M_p(Z,A) = -\frac{1}{2}\alpha_3 M_0^2 \frac{(-1)^Z + (-1)^{A-Z}}{R(A)}, \tag{7}$$

which depends on the even or odd value of the proton number or neutron number, similar to the nuclear droplet model, the parameter α_3 is a fitting parameter.

The mass formula was fitted to the experimental atomic masses by computer and the fitting parameter values were obtained. In order to improve the mass calculation inaccuracy of light nuclei, the radiation term was modified

$$M_{rad}(A) = -C_{rad}(A-1.5)^2 M_0^2 / R(A),$$
$$R(A) = B^{\sqrt{(A-1.5)M_0}} - 1, \ (A \geq 2). \quad (8)$$

The Nuclear Radiation Model was fitted to approximately 2000 experimental atomic masses (isotopes), using the publication of *G. Audi* and *A. H. Wapstra* [1]. Fitting the NRM meant determining two parameters. One of the fitting parameters is the mass of the neutron itself, the result of the fitting

$$M_n^* = 9.394120054E2 \ MeV / c^2, \quad (9)$$

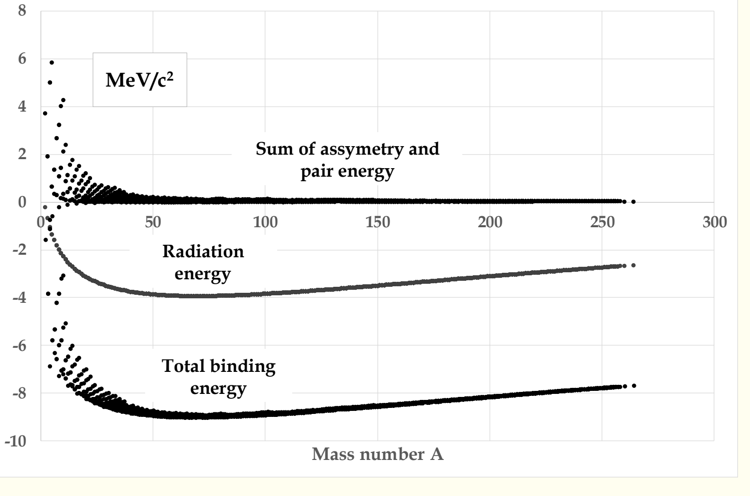
what is about the experimental mass of the neutron. The other fitting parameter

$$Q^* = 0.223481204... \cong 2/9 = Q. \quad (10)$$

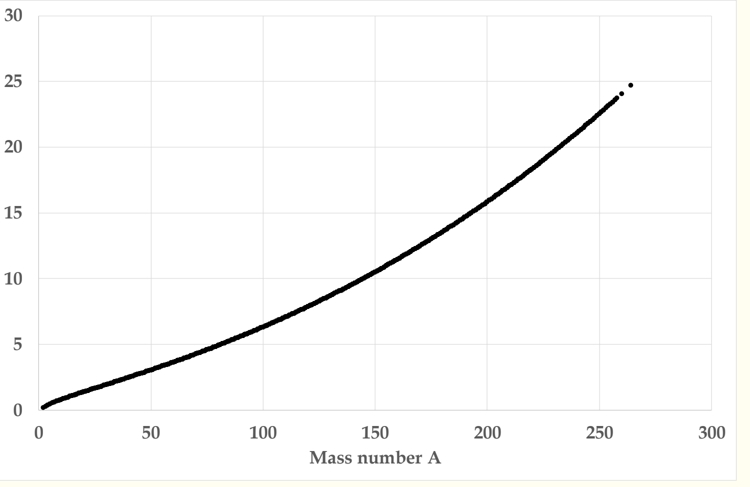
The alpha fitting parameters are clearly defined by the number Q^* (see later). The fitting parameter *B* is

$$B = 1 + \alpha_0 = 1.223481204..., \ (\alpha_0 \equiv Q^*). \quad (11)$$

In summary, the fitting of the NRM model means the fitting of only two parameters given the experimental atomic masses, these are the neutron mass and the number $Q \approx 2/9$. The calculation was done with the MS Visual Basic program on a personal computer.



Graph 1. Energy components per nucleon in the NRM model as a function of the mass number *A.*



Graph 2. The calculated nuclear radii (in proportion)
in the NRM model as a function of the mass number *A.*

The accuracy of the NRM model is characterized by the relative standard error

$$\sigma = \sqrt{\frac{1}{n-1}\sum_i \left(\frac{M_{i,calc} - M_{i,\exp}}{M_{i,\exp}}\right)^2} = 1.512\ldots \times 10^{-4}, \tag{12}$$

where n is the number of atoms taken into account. In a previous calculation, the nuclear liquid droplet model based on the same atomic mass data resulted similar accuracy. Graphs show the special results of the new atomic mass calculation model.

The fusion temperature

In parallel with the fitting of the NRM, the fusion temperature required for the complete periodic table of atoms is also determined. Eq. (8) defines the radius of the nucleus, which is closely related to the formula for the average oscillator energy of nucleons

$$R(A) = B^{\sqrt{(A-1.5)M_0}} - 1 = (1+Q)^{\sqrt{(A-1.5)M_0}} - 1. \tag{13}$$

Compared to Planck's radiation law

$$(1+Q)^{\sqrt{M_0}} = \exp(\hbar\omega_0 / kT_{fus}) = \exp(M_0 / kT_{fus}), \tag{14}$$

from which

$$T_{fus} = \sqrt{M_0} / k \ln(1+Q) = 5.367\ldots E + 13\ K, \tag{15}$$

i.e. about 54 billion degrees Kelvin. This is the temperature required for the fusion of the atoms of the entire periodic table.

Important:

- The alpha fitting parameters are calculated from the fitted value of *Q.*
- In the calculations, both the atomic masses and the mass of the neutron are given in atomic units (a.u.), which are *12th* of the mass of the C-12 atom.

3. The Quantized Mass Oscillator

The birth of exact quantum mechanics in 1925 is attributed to Werner Heisenberg [2], who provided the first example of quantized form of the classical harmonic oscillator

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega; \quad (n = 0,\ 1,\ 2,\ldots). \tag{16}$$

According to the theory of relativity, the following equation is satisfied for the four-vector of the EM field

$$p_\mu p^\mu = 0 \;\Leftrightarrow\; E^2 - P^2 = 0, \;\left(c^2 = 1\right). \tag{17}$$

This equation can also be written in the following form

$$\begin{aligned} &\left(E - a_{n+1}\right)^2 = \left(P - a_n\right)^2 = \left(E - a_n\right)^2; \\ &a_n = n\hbar\omega,\; a_{n+1} = \left(n+1\right)\hbar\omega; \;(n = 0,\ 1,\ 2,\ldots), \end{aligned} \tag{18}$$

which equations define the energies of the quantized electromagnetic field packages

$$E_n = \frac{a_n + a_{n+1}}{2} = \hbar\omega\left(n + \frac{1}{2}\right); \;(n = 0,\ 1,\ 2,\ldots). \tag{19}$$

The quantization of particles with non-zero rest mass is done similarly to above method. The four-momentum equation of the rest particle

$$p_\mu p^\mu \Rightarrow p_0 p^0 = E^2 - P^2 = m_0^2 > 0, \tag{20}$$

where m_0 is the rest mass of the particle, E is the self-energy and P is the self-momentum of the particle, by definition.

The particle quantization is similar to the quantization of the EM space

$$E_n^2 - P_n^2 = \left(E - a_{n+1}\right)^2 - (P - a_n)^2 = m_0^2; \;(n = 0,\ 1,\ 2,\ldots), \tag{21}$$

where the mass components are monotonically increasing

$$\lim_{n\to\infty} E_n^2 = E^2, \;\lim_{n\to\infty} P_n^2 = P^2. \tag{22}$$

Definition of zero-point energy of the particle

$$E_0^2 = \left(E - a_1\right)^2 = m_0^2 \;\Rightarrow\; \left(P - a_0\right)^2 = 0. \tag{23}$$

The first excitation level of the quantized particle

$$E_1^2 - P_1^2 = \left(E - a_2\right)^2 - \left(P - a_1\right)^2 = m_0^2, \tag{24}$$

which can be calculated from the previous equation.
Similarly, we get the additional energy levels (recursion formula)

$$E_n^2 - P_n^2 = \left(E - a_{n+1}\right)^2 - \left(P - a_n\right)^2 = m_0^2, \ (n = 0,\ 1,\ 2, \ldots). \qquad (25)$$

The *Quantized Mass Oscillator* (QMO) is a natural generalization of the Quantized Harmonic Oscillator (QHO) from the Quantum Mechanics. The QMO, shortly "mass oscillator" is defined by two parameters, the mass of the particle and the universal Q dimensionless constant. The value of the Q parameter is independent of the size of the particle masses, its nominal value is $Q = 2/9$.

For the application of the mass oscillator in particle physics, we have to introduce two axioms

AXIOM I.

The quantized structure of every particle (26)
is the same regardless of its mass magnitude.

AXIOM II.

For every mass quantized particle (27)
the structure ratio is same: $P / m_0 = Q = 2/9$.

It is advisable to introduce the normalized mass oscillator i.e. NQMO

$$\varepsilon_n^2 - \pi_n^2 = \left(1 - \alpha_{n+1}\right)^2 - \left(Q - \alpha_n\right)^2 = \mu_0^2, \qquad (28)$$

where

$$\varepsilon_n = \left(E - a_{n+1}\right)/m_0,\ \pi_n = \left(P - a_n\right)/m_0;$$
$$\alpha_0 = Q,\ \mu_0 = \sqrt{1 - Q^2},\ E \equiv m_0,\ (n = 0,\ 1,\ 2, \ldots). \qquad (29)$$

The first few alpha values of NQMO are here ($Q = 2/9$)

$$\begin{aligned}
\alpha_8 &= 6.240509992E-07 \ \cong\ 6.607807467E-07 = 0.5 \times Q^9 \\
\alpha_7 &= 2.808246364E-06 \ \cong\ 2.973513360E-06 = 0.5 \times Q^8 \\
\alpha_6 &= 1.263745023E-05 \ \cong\ 1.338081012E-05 = 0.5 \times Q^7 \\
\alpha_5 &= 5.687544504E-05 \ \cong\ 6.021364554E-05 = 0.5 \times Q^6 \\
\alpha_4 &= 2.560797723E-04 \ \cong\ 6.021364554E-05 = 0.5 \times Q^5 \\
\alpha_3 &= 1.155214096E-03 \ \cong\ 1.219326322E-03 = 0.5 \times Q^4 \\
\alpha_2 &= 5.257657428E-03 \ \cong\ 5.486968450E-03 = 0.5 \times Q^3 \\
\alpha_1 &= 2.500395696E-02 \ \cong\ 2.469135802E-02 = 0.5 \times Q^2 \\
\alpha_0 &= 2.222222222E-01 \ =\ 2.222222222E-01 = Q = 2/9
\end{aligned} \qquad (30)$$

Table 1. Some lower levels of the normalized mass oscillator (NQMO).

These alpha values first were used in the introduced Nuclear Radiation Model.
The above equations give uniform structure of every existing elementary particles.

4. A unified theory of physical interactions

According to today's knowledge, there are four independent forms of physical interaction: gravity, electromagnetism, weak and strong interaction. In the meantime, it was possible to find the common root of electromagnetism and the weak interaction, which is now called the “electroweak interaction”, thus reducing the four physical interactions to three types of interactions that are now thought to be independent. The Standard Model, representing the pinnacle of today's theoretical physics, includes the electroweak and strong interactions in their quantized form.

The firm belief of the Author of the work, the fundamental interactions of physics are not independent, each physical interaction operates according to the same physical-mathematical principle. According to our universal theory, all physical interactions can be traced back to changes in the self-momentums of the participating physical objects. Most of the interactions involve changes in the interacting masses, namely the emission and absorption of massless objects (photons, neutrinos). The weak interaction involves a change in mass and at the same time a change in charge: electron, muon, or tau particle emission or absorption. The same can be said about pair-creation and scattering processes (particle-antiparticle interaction).

The energies of massless particles (photon, neutrino) are equal to their self-energies corresponds to their self-momentums. In the case of gravity and electromagnetic interactions, the change in the self-momentum masses of the interacting objects is negligible compared to their rest masses.

During the weak and strong interactions known from nuclear physics, the changes in the self-momentum of the participating objects are significant compared to their rest masses. For the theoretical description of weak and strong interactions, we use the Quantized Mass Oscillator (QMO) of the material particles. In the calculations, in each interaction process, we calculate the starting state and final state masses. The kinetic energy generated or absorbed in the processes is also counted in the balance of the self-momentum of the process, as are the energies of the generated or absorbed photons and neutrinos.

The mass spectrum of each particle is the same; the Quantized Mass Oscillator. During emission or absorption, when the mass emits or absorbs a photon (neutrino), the mass change of the particle is expressed by the following multiplication factors

Emission

$$m' \cong m \cdot \left(1-\alpha_j\right) \equiv m \cdot \beta_j; \ \left(j=1,\ 2,\ 3,\ldots,\ n\right), \tag{31}$$

Absorption

$$m \cong m' / \left(1-\alpha_k\right) \equiv m' \cdot \beta_k^{-1}; \ \left(k=1,\ 2,\ 3,\ldots,\ n\right). \tag{32}$$

The alpha series is defined by the NQMO model introduced above. The first few members of the alpha series are defined by Table 1. The higher the *k* values, the more accurate the approximate equations are. For nuclear transformations, *k >= 7* provides a sufficiently high accuracy. In multistage processes, the mass of the given particle either continuously increases or continuously decreases, the mass structure of the resulting particles after each step will be similar to the previous one.
The multistep emission processes

$$\begin{gathered} M_{OUT} = \beta_{k_1}^{i_1} \cdot \beta_{k_2}^{i_2} \ldots \cdot \beta_{k_n}^{i_n} M_{IN}; \\ \left(i_1,\ i_2,\ldots,i_n = 0,\ 1,\ 2,\ 3,\ 4;\ k_1,\ k_2,\ldots,\ k_n = 1,\ 2,\ 3,\ldots,\ n\right), \end{gathered} \tag{33}$$

where M_{IN} is the sum of the masses before the interaction and M_{OUT} is the sum of the masses after the interaction. In the emission processes the *i* indices are positive values, in absorption processes the *i* indices are negative numbers.

In the case of nuclear physics processes, the change in the self-momentum of the particles is significant, therefore the nuclear processes are described by the multistep products presented here. The individual processes are described with square brackets in abbreviated form

$$M_{IN} \Rightarrow \left[\beta_1^{i_1},\ \beta_2^{i_2},\ \beta_3^{i_3},\ ...,\beta_n^{i_n}\right] \Rightarrow M_{OUT}. \qquad (34)$$

In the calculations, we use the nominal value *Q = 2 / 9*. In all cases, the relative error of the calculation is given, denoted by δ.

5. The Strong Interaction

According to today's knowledge, the fusion of atoms only takes place at extremely high temperatures, when stars explode (supernovae, merging black holes). The fusion of atomic nuclei occurs in pairs, atoms with a smaller mass transform into nuclei with a larger mass during their collision, and huge energy is released in the process. On the other hand, the fusion of atomic nuclei heavier than iron 56 isotopes requires energy investment, for which high-temperature radiation provides the energy. As is well known, the planned future nuclear energy production will be based on the fusion of light nuclei, the current objective is to achieve the fusion of hydrogen atoms, which is the dominant part of the Sun's radiation energy.

In the initial successful hydrogen bomb experiments, the explosion of a smaller uranium bomb ensured the required fusion temperature, the bomb's mantle was filled with a mixture of deuterium and tritium (isotopes of the hydrogen atom). During fusion, He-4 atoms are formed and neutrons are released. A lot of energy is released in the process

$$^2_1\mathrm{D} + {}^2_1\mathrm{T} \Rightarrow {}^4_2\mathrm{He} + \mathrm{n} + 14.06\ MeV / c^2. \qquad (35)$$

The QMO model describes the fusion of nuclei, usually the transformation of atomic nuclei, as a multistep process. Each atomic nucleus forms a QMO, the energy spectrum of which is determined by its recursion formula. The energy of the photons that can be emitted or absorbed by the mass oscillator is an exponentially decreasing infinite series, which roughly corresponds to integer powers of the number *Q = 2 / 9.*

The sum of the photon energies gives the total internal self-energy of each atomic nucleus. While the energy spectrum of the QHO is the sum of constant energies, it is not possible to distinguish which quantum of energy the photon oscillator emits. The energies of the spectrum of the atomic nucleus as a mass oscillator can be distinguished, and therefore can also be numbered on the set of natural numbers.

At a high-temperature cluster of atomic nuclei, the emission and absorption of photons in the range of high-energy quanta is likely. Obviously, the radiation of higher-energy quanta is important for the nuclear synthesis, such as the above-mentioned thermonuclear fusion, the process can only take place at very high temperatures. After a nucleus emits a quantum of energy, its mass decreases, the end result is also a mass oscillator, also determined by the parameter *Q*. The emissions follow each other one after the other until to the final state is realized.

If the structure of the mass oscillator spectra depended on the size of the masses, it would add a large amount of specific information to the different masses, what we exclude based on the principle of simplicity of physics.

The D + T process in the QMO model is described by the following symbolic notation (δ is the relative error of the calculation)

$$\mathrm{D + T \Rightarrow He\text{-}4 + n} \Rightarrow \left[\beta_3^3, \beta_4, \beta_6^2, \beta_7\right], \ (\delta = 7.4E\text{-}07). \tag{36}$$

The essence of the QMO nuclear physics calculation lies in the assumption that the energy levels of a mass oscillator assigned to a mass of any size differ from each other only by a proportionality factor, the proportionality factor being the intermediate, momentarily existing mass.

The D + T fusion process is realized quickly, the lifetime of intermediate particles is very short, we can consider them as "virtual" particles known from quantum electrodynamics. The atoms of the complete periodic table are created through a series of two-particle collisions at the very high fusion temperature.

Fusion of deuterium

$$\mathrm{n + p} \Rightarrow {}_1^2\mathrm{D} + 2.22452\ MeV/c^2. \tag{37}$$

The mass oscillator model

$$\mathrm{n + p + e^-} \Rightarrow {}_1^2\mathrm{D} \Rightarrow \left[\beta_3, \beta_6^2, \beta_7\right], \ (\delta = 1.24E\text{-}9). \tag{38}$$

The result agrees with experience, deuterium does not have significant excited states (more precisely, they are negligible).

A possible fusion of chlorine Cl-36 (e^+ = positron)

$${}_{24}^{12}\mathrm{Mg} + {}_{12}^{6}\mathrm{C} \Rightarrow {}_{36}^{17}\mathrm{Cl} + e^+ + 15.077\ MeV/c^2. \tag{39}$$

The symbolic form of the process is

$$\mathrm{M_{IN}} \Rightarrow \left[\beta_4, \beta_5^3, \beta_6, \beta_7^2\right] \Rightarrow \mathrm{M_{OUT}} \ (\delta = 7.1E\text{-}7). \tag{40}$$

From the point of view of fusion, in this case the *1 : 2* mass ratio is ideal. It is known that during the fission of uranium, the mass ratio of the most common fragments is around *1 : 2*. It can be assumed that in the reverse of this process, the nuclear fusion also occurs with the greatest probability at this mass ratio.

According to the above, the process of nuclear fusion takes place through several steps, exactly through the possible excitation states of the resulting nucleus. The gamma radiation spectrum of the process can in principle be easily measured with today's modern instruments (semiconductor detectors), so the theory can be supported experimentally.

6. Conclusion

Since the beginning of the twentieth century, with the birth of quantum physics, the central endeavor of modern physics has been the unification of the separate theories of light and matter. The quantized nature of the electromagnetic field at the level of the microworld is precisely described by quantum mechanics, which was discovered and developed to a high level in the first quarter of the twentieth century. The most important field of application of quantum mechanics is atomic and molecular physics. At the same time, the principles of quantum mechanics also led to the understanding of solid-state physics, semiconductor physics and superconductivity.

In the meantime, it turned out that the theorems of quantum mechanics are also valid in the physics of the atomic nucleus. According to today's physics, the one possible exact description of the atomic nucleus can only take place within the framework of a many-particle model (and of course, only with the formalism of quantum mechanics). It also turned out that the building blocks of the atomic nucleus, protons and neutrons, can be further divided into quarks, which seems to be supported by numerous theoretical and experimental facts. What is very disturbing in this regard is the fact that free quarks have not yet been detected, which is why the theoretical explanation was born: free quarks cannot exist by even theoretically. The binding of the atomic nucleus is held together by the strong interaction assumed long ago, the current form of which is the QARK-QCD theory.

In parallel, there are currently two simple physical models exist for the approximate determination of the ground state masses of atomic nuclei, which are single-particle models. The one is the old and still successfully used Bethe-Weizsäcker model [3], which considers the nucleus as a classical liquid droplet (SEMF). The other single-particle nuclear model is the newest Nuclear Radiation Model (NRM), developed by the Author of this paper.

The discovery of the electron, the muon, and a large number of "elementary particles" unique show the general quantized nature of matter. The quantum nature of the electromagnetic field was proven long ago with the Quantized Harmonic Oscillator model (QHO). The mass quantization of particles with non-zero rest mass realized for the first time by the Author of this paper with the introduction of the Quantized Mass Oscillator model (QMO). This oscillator model is theoretically suitable for describing all types of known physical interaction.

Nuclear fusion takes place step by step during the emissions of successive mass oscillators with continuously decreasing the masses of the nuclei. The presented calculation method is not only suitable for calculating not only the mass of the ground state nuclei, but it can also be used for calculation of excited states of these nuclei. These calculations capable provide structural information on the individual quantum states of the nuclei, and detailed information from the fusion process. All this makes it possible to analyze and plan the different types of nuclear reactions in the future.

This new nuclear calculation model, for example, would be an important testing tool of the followed closely fusion power plant research in France "ITER" [4]. It can be assumed that the QMO can be successfully extended to the investigation of high-energy elementary particles as well.

Unfortunately, it is an undeniable fact that today's physics cannot interpret the actual functioning of the atomic nucleus. So far, many years of unsuccessful research have led the researchers to accept that the operation of the atomic nucleus is so complicated that the previously known theoretical methods of physics are unsuitable for understanding the atomic nucleus and nuclear processes. There was no other solution but to invent a very complicated physical-mathematical theory, this is now the exotic KVARK-QCD theory.

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